Problem 1: Fermat Theory - Solution

Assume $f(x) = \sum_{k=0}^{n} a_k x^k$. Let us start by working out $f(x+y)$ using Newton's binomial theorem.

$$
\begin{aligned}
f(x+y) &= \sum_{k=0}^{n} a_k (x+y)^k \\
&= \sum_{k=0}^{n} \sum_{l=0}^{k} C_l^k a_k x^l y^{k-l} \\
&= \sum_{k=0}^{n} C_k^k a_k x^k + \sum_{k=1}^{n} \sum_{l=0}^{k-1} C_l^k a_k x^l y^{k-l} \\
&= \sum_{k=0}^{n} a_k x^k + y \sum_{k=1}^{n} \sum_{l=0}^{k-1} C_l^k a_k x^l y^{(k-1)-l} \\
&= f(x) + y h(x,y)
\end{aligned}
$$

So, we see that $g(x) = f(x)$. Now, we have two options: one is to note that

$$
h(x,y) = \frac{f(x+y) - f(x)}{y}.
$$

Thus, we see $h(x,0) = \lim_{y \to 0} h(x,y) = f'(x)$. The other option is to note that

$$
\begin{aligned}
h(x,0) &= \sum_{k=1}^{n} C_{k-1}^k a_k x^{k-1} \\
&= \sum_{k=1}^{n} k a_k x^{k-1} = f'(x)
\end{aligned}
$$

So, to implement the solution of this problem we simply have to multiply the numbers in the list by their index, apply the right modulo operation and print the last $n-1$ elements of the list. And watch the edge case of $f(x)$ being constant.